

If $\vec{A}, \vec{B}, \vec{C}$ are ~~two~~ three vectors then

$$\frac{d}{dt} (\vec{A} \cdot \vec{B} \times \vec{C}) = \frac{d\vec{A}}{dt} \cdot \vec{B} \times \vec{C} + \vec{A} \cdot \frac{d\vec{B}}{dt} \times \vec{C} + \vec{A} \cdot \vec{B} \times \frac{d\vec{C}}{dt}$$

Proof:

$$\begin{aligned} \frac{d}{dt} (\vec{A} \cdot \vec{B} \times \vec{C}) &= \frac{d\vec{A}}{dt} \cdot (\vec{B} \times \vec{C}) + \vec{A} \cdot \frac{d}{dt} (\vec{B} \times \vec{C}) \\ &= \frac{d\vec{A}}{dt} \cdot \vec{B} \times \vec{C} + \vec{A} \cdot \left(\frac{d\vec{B}}{dt} \times \vec{C} + \vec{B} \times \frac{d\vec{C}}{dt} \right) \\ &= \frac{d\vec{A}}{dt} \cdot \vec{B} \times \vec{C} + \vec{A} \cdot \frac{d\vec{B}}{dt} \times \vec{C} + \vec{A} \cdot \vec{B} \times \frac{d\vec{C}}{dt} \end{aligned}$$

Ex: 1 Let \vec{A} be a vector find

$$\frac{d}{dt} \left(\vec{A} \cdot \frac{d\vec{A}}{dt} \times \frac{d^2\vec{A}}{dt^2} \right)$$

(2) Prove that $\vec{A} \times \frac{d^2\vec{B}}{dt^2} - \frac{d^2\vec{A}}{dt^2} \times \vec{B}$

$$= \frac{d}{dt} \left(\vec{A} \times \frac{d\vec{B}}{dt} - \frac{d\vec{A}}{dt} \times \vec{B} \right)$$